Exercise 2C

1 a $x^2 + 4x = (x + 2)^2 - 2^2$
$= (x + 2)^2 - 4$

b $x^2 - 6x = (x - 3)^2 - 3^2$
$= (x - 3)^2 - 9$

c $x^2 - 16x = (x - 8)^2 - 8^2$
$= (x - 8)^2 - 64$

d $x^2 + x = \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$
$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$

e $x^2 - 14 = (x - 7)^2 - 7^2$
$= (x - 7)^2 - 49$

2 a $2x^2 + 16x = 2(x^2 + 8x)$
$= 2((x + 4)^2 - 4^2)$
$= 2((x + 4)^2 - 16)$
$= 2(x + 4)^2 - 32$

b $3x^2 - 24x = 3(x^2 - 8x)$
$= 3((x - 4)^2 - 4^2)$
$= 3((x - 4)^2 - 16)$
$= 3(x - 4)^2 - 48$

c $5x^2 + 20x = 5(x^2 + 4x)$
$= 5((x + 2)^2 - 2^2)$
$= 5((x + 2)^2 - 4)$
$= 5(x + 2)^2 - 20$

d $2x^2 - 5x = 2\left(x^2 - \frac{5}{2}x\right)$
$= 2\left(\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right)$
$= 2\left(\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right)$
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8}$

e $8x - 2x^2 = -2x^2 + 8x$
$= -2(x^2 - 4x)$
$= -2((x - 2)^2 - 2^2)$
$= -2((x - 2)^2 - 4)$
$= -2(x - 2)^2 + 8$

3 a $2x^2 + 8x + 1 = 2(x^2 + 4x) + 1$
$= 2((x + 2)^2 - 2^2) + 1$
$= 2(x + 2)^2 - 8 + 1$
$= 2(x + 2)^2 - 7$
So $p = 2$, $q = 2$ and $r = -7$

b $5x^2 - 15x + 3 = 5(x^2 - 3x) + 3$
$= 5\left(\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) + 3$
$= 5\left(x - \frac{3}{2}\right)^2 - \frac{45}{4} + 3$
$= 5\left(x - \frac{3}{2}\right)^2 - \frac{33}{4}$
So $p = 5$, $q = \frac{3}{2}$ and $r = -\frac{33}{4}$

c $3x^2 + 2x - 1 = 3\left(x^2 + \frac{2}{3}x\right) - 1$
$= 3\left(\left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) - 1$
$= 3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} - 1$
$= 3\left(x + \frac{1}{3}\right)^2 - \frac{4}{3}$
So $p = 3$, $q = \frac{1}{3}$ and $r = -\frac{4}{3}$

d $10 - 16x - 4x^2 = -4x^2 - 16x + 10$
$= -4(x^2 + 4x) + 10$
$= -4((x + 2)^2 - 2^2) + 10$
$= -4(x + 2)^2 + 16 + 10$
$= -4(x + 2)^2 + 26$
So $p = -4$, $q = 2$ and $r = 26$

e $2x - 8x^2 + 10 = -8x^2 + 2x + 10$
$= -8\left(x^2 - \frac{1}{4}x\right) + 10$
$= -8\left(\left(x - \frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2\right) + 10$
$= -8\left(x - \frac{1}{8}\right)^2 + \frac{1}{8} + 10$
$= -8\left(x - \frac{1}{8}\right)^2 + \frac{81}{8}$
So $p = -8$, $q = -\frac{1}{8}$ and $r = \frac{81}{8}$

4 $x^2 + 3x + 6 = \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 6$
$= \left(x + \frac{3}{2}\right)^2 + \frac{15}{4}$
$a = \frac{3}{2}$ and $b = \frac{15}{4}$

5 $2 + 0.8x - 0.04x^2 = -0.04x^2 + 0.8x + 2$
$= -0.04(x^2 - 20x) + 2$
$= -0.04((x - 10)^2 - 10^2) + 2$
$= -0.04(x - 10)^2 + 4 + 2$
$= 6 - 0.04(x - 10)^2$
$A = 6$, $B = 0.04$ and $C = -10$